

The Father of Long/Fat Tails Interview with Benoît Mandelbrot

HANS ULRICH OBRIST

Hans Ulrich Obrist: I wanted to ask you when your interest in science actually started, if one can localise the beginnings.

Benoît Mandelbrot: I had the fortune and the misfortune of being the nephew of a very well known mathematician. When I was thirteen, my uncle became professor at the Collège de France, and so I always knew that becoming a scientist was an option. But what kind of scientist? A decision taken in the middle of the war, under conditions that were quite dreadful in many ways, has marked me for life. First of all, in January 1944 (I remember very well how it happened) I realised that my gift for shape and geometry was truly extreme. When the professor read the problem that was a question of algebra or of analysis, I did not hear formulas, but saw pictures.

HO: That was in Lyon?

BM: Yes. Clearly, you have read a great deal about me! Geometry came easily, a good reason for loving it. Second, it provided me with a competitive advantage; in my generation, nobody else had this gift to the same degree. Another advantage: this gift does not fade with age. All my old friends complain that they find it increasingly difficult to sit down and write new formulas. But the ability to look for structure in pictures where everybody else sees a mess does not decline with age as much. That may help explain why, past eighty, I still work effectively almost full-time.

I also made another discovery about myself: I deeply wanted to identify some field where I could emulate Kepler ; not Newton [laughs], not Copernicus, but Kepler. I put an extremely high value on the precise moment when an idea which is very abstract and

removed from any applications ; call it a *toy* ; is made over into a *tool* and used to understand part of the messiness of nature. I constantly search for situations where a toy can become a tool. For example, I like to design my books myself. I find it particularly fulfilling to start with an idea and contribute to every step till it has become a book.

HO: Like the book you wrote with Michael Frame in 2002, *Fractals, Graphics and Mathematics Education*?

BM: In that book we had some freedom of design, but not enough, because the publisher has a very specific format that I didn't choose.

HO: There are books that are completely designed by you, like artists' books almost.

BM: *The Fractal Geometry of Nature*, which came out in 1982 and is my major work, was entirely designed to my precise specifications, including every picture. The official designer became a good friend and prefers to describe himself as the co-designer because he made the pictures better in quality and size, but followed very precisely what I wanted.

HO: It is very interesting that in terms of the art world images are important, and very often in science there is an opposite approach, of not trusting images. You told previous interviewers that in Lyon, when you had your first experience of your mathematical skills, images played a very big role. I was wondering if you could tell me about this importance of images, which is a little bit against the grain of what most scientists do.

BM: It certainly stands completely against the ideology of the 1960s and '70s, when the sciences were sharply classified according to whether images are or are not important. A German-born friend of mine, a great biologist and philosopher, went so far as to theorise that progress in science consists in eliminating pictures as much as possible. Mathematics was perfect because it had completely banished pictures^a even from elementary textbooks. I put the pictures back. This was received in a very hostile fashion by most of my colleagues. Since then, the opposition to pictures has weakened, simply because they have been so extraordinarily fruitful and because humans are continually changing.

Scientists are not separate from their society and technology. When I was a child, books had no pictures because of economics: pictures were expensive; so books were very dry and grey by design. Russian books, almost until the fall of the Soviet Union, remained extremely grey, whereas now the environment in which everybody is raised is extremely colourful, rich in design. The young mathematicians cannot help being more open to the influence of pictures, but nobody claims that pictures are as important to his or her work as to mine.

HO: In the art world, there have been moments like the Black Mountain College with John Cage and Buckminster Fuller that have been amazing realisations of trans-disciplinarity; such moments usually produce the most interesting expressions of art and architecture. In terms of your education, you had on the one hand the École Polytechnique, a kind of trans-disciplinary school, but you had also this practical experience with IBM, where a lot of disciplines formed a pool of knowledge. In the current world of art and

architecture, this fear of pooling knowledge is still very widespread. Could you tell me about these experiences?

BM: IBM was a quite an experience, but more important was my age during World War II, and the effects of moving from Poland to France and then to the US. In many ways, I was very under-educated or mis-educated; for years I was either very much older than my classmates, or I was with the class but not influenced by it. During my early years my mother did not let me go to school, but had me tutored by an uncle, not the famous one, who did not teach me anything in a dogmatic fashion. The various schools I went to didn't put a strong stamp on me.

However, the IBM I knew for 35 years no longer exists. It is now a very large corporate development centre concerned with fulfilling Wall Street's and the stockholders' expectations. Earlier, IBM felt responsible for the whole development of the computer, and also had an extremely enlightened policy of enabling creative freedom, contingent on company approval. This policy has been extremely fruitful. It was a very special period; I am very glad to have stumbled there when it was beginning, and to have stayed for 35 years.

HO: Did you have exchanges there with other scientists?

BM: Exchange is something one chooses to have or not. I have written and co-authored many articles and books with people in my own field, and also in other fields where I have noticed something peculiar with regard to an image, and felt the need to quantify that.

HO: You were one of the very last students of the mathematician John von Neumann. Can you tell me about this?

BM: I never worked with von Neumann in the sense of asking him for suggestions. I am very pleased to have spent some time with him, though our methods were not compatible. He worked in logic, then in physics, economics, pure mathematics and applied mathematics. Logic and the computer were impressively integrated, but the other fields, in the main, had very little in common, so his other works might have been done by different persons without losing particular substance. To the contrary, all my work involves one theme or a few themes very closely linked together and could be viewed as contributing to a single highly interdisciplinary topic.

HO: Albert Hoffman, the inventor of LSD, told me in an interview that chance played a big role in his discovery. How did you discover fractals, and was chance a crucial factor, as in the case of Hoffman?

BM: Completely. Early in my life, I was enormously influenced by a very short article in *Scientific American* that my uncle received from the author, his colleague Joseph Walsh. He himself was not interested and had thrown it into a wastebasket. He took it out, and gave it to me. The article was a review of a book by George Kingsley Zipf, concerning the distribution of word frequencies. I was fascinated. The question arises: what if I hadn't gone to see my uncle that day, or if he had not thrown the article into the wastebasket, or if it had not been sent to him?

Later in life, an equally important influence was exerted by a diagram I happened to see by chance on a dirty blackboard at Harvard, when I was going to lecture.

HO: Some drawing from a previous lecture?

BM: Yes. I saw a very specific shape with alternating convex and concave portions. I looked at the blackboard, then at my host, and asked him to explain. He did so, saying the topic was entirely different from my work. But the diagram proved that in fact it was very close. That chance event provoked me to study financial markets, and resulted in 1963 in a paper titled "The Variation of Certain Speculative Prices". It became extremely famous and very influential, a *Citation Classic*, especially then and now. The story is told in a book I co-authored with the journalist Richard Hudson, *The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward*, which came out in 2004.

These two chance events raise a serious question. The blackboard might have been cleaned before I came in, or made so dirty by overwriting that this drawing would not have been visible. Things of this sort affect a life.

"What would have happened?" Quite possibly, neither phenomenon would have been noticed by anybody; the delay, scientifically speaking, might have cost 40 years in the case of the study of prices, because for 40 years the underlying data was known but not taken seriously, and dismissed. Or my work in this area might have been postponed even further because the conditions which interest so many people might not have changed. The book I mentioned might not have been written at all.

An odd thing is that chance has also helped me on many other occasions. Louis Pasteur is credited with the observation that chance can only help the well-prepared mind. I also think that my long string of lucky breaks can be credited to my mode of paying attention: I look at funny things and never hesitate to ask questions. Most people would not have noticed the dirty blackboard, or looked at the article that my uncle gave me because he was not interested.

Let me add a comment. That reprint in 1951 and that diagram on the chalkboard both concerned examples of what are now called long-tailed or fat-tailed distributions. These evanescent episodes made me the first well-trained mathematician to take those tails seriously. For this reason, I have heard myself being called "the father of long tails". Long η or fat η tails are an intimate part of the fractal family, so that term and the more common alternative "the father of fractals" do not contradict each other.

HO: With regard to those two chance events, you remarked in an earlier interview that the connection between them appeared wild to you on the first night, but by the second night, you had become accustomed to it. Can you tell me about that moment?

BM: I couldn't do anything about my intuitive speculations until the computer became available and I decided to make it into a tool of experimental mathematics. Observe that my best-known discovery was not due to the availability of exceptionally good pictures at IBM. Quite to the contrary, my best work was performed in 1980. While I was a visiting professor at Harvard, I had to deal with complicated research conditions within a very bad system. The pictures seen on the first night seemed incomprehensible; the second night, they became more coherent. Within a few days they had become completely familiar, as though one had always seen them. Incredible! The set to which

they belonged soon became known as the Mandelbrot Set, and you know how popular it became. But early on, most of the mathematicians who followed my work didn't like the idea of a mathematical discovery based on a picture. All that is discussed in detail in my book *Fractals and Chaos: The Mandelbrot Set and Beyond*, which Springer published in 2004.

HO: Artists and architects have always been interested in this idea of the multi-fractal in relationship to the phenomenon of turbulence. When did this element enter your work?

BM: I didn't realise that architects and painters were interested in multi-fractals, though I knew of their interest in ordinary fractals, uni-fractals. But I am not surprised. It had to come. In 1963, a paper I had written with J.M. Berger, titled "A New Model of Error Clustering on Telephone Circuits", was published in the IBM Journal of Research and Development. It looked very technical but was in fact very mysterious. A bit earlier, during the Cold War, the great Soviet scientist Andrei Kolmogorov had published in *The Journal of Fluid Mechanics* (and other places), an article that became famous, but which I found impossible to understand. I thought I could understand Kolmogorov through the paper I co-authored with Berger, but soon realised that this was the wrong path. It took me several years of extremely hard work to conclude that Kolmogorov had made a mistake. It is still very difficult to state this about a man of his brilliance. What he published was mathematically wrong, without any doubt. By re-doing it properly, I developed a multi-fractal model that addressed the intermittence of turbulence but also has turned out to be fundamental to our understanding of the variation of financial prices. All that is retold in my book *Multifractals and 1/f Noise: Wild Self-Affinity in Physics (1963-1976)*, published by Springer in 1999.

HO: The late artist Alighiero Boetti conceptualised systems of order and disorder in which the order also simultaneously implies a disorder. For instance, he compiled a list of one thousand of the world's longest rivers, published in a book in 1977^a. Obviously, there is no absolutely fixed length of river, or a single reliable source, there are multiple and varying sources. This project involved immense geographic and scientific measurements, but with a preordained ambiguity in the results. Is this different from your notions of order and disorder?

BM: The general observation, that the different sources of rivers account for their different lengths, had already been made by hydrologists. The specific explanation I gave in 1967 concerned the fact that if techniques explicitly designed to study order are applied to disorder, the results will demand careful re-examination. Observe that order and disorder are distinctions at the level of mechanisms, structure, interactions in the system. I like to focus on the basic pattern.

A primitive man or woman saw very few, simple, smooth shapes. For example, the full moon is a simple shape, a circle. The pupil and the iris of the eye are circles. Some berries are spherical. But in the wild, almost all the shapes are extremely rough and complicated; there is a sharp distinction between the smooth/simple and the rough/complicated. Historically, geometers concentrated on the properties of a very

few smooth shapes and physicists were also significantly devoted to smooth, regular behaviour, with perhaps sometimes a complication of the kind that the French mathematician René Thom theorises as "catastrophe". But trees are not smooth at all, neither are mountains and clouds.

A remarkably large number of artists had no vocabulary to express their grasp of the nature of fractals, yet such an understanding comes through very clearly in their work.

HO: You have frequently cited the Japanese artist Hokusai as an example of this sensitivity.

BM: Hokusai was at his peak around 1800^a almost our contemporary. But history provides examples of many earlier painters or philosophers who were very aware of complicated shapes with fractal structure. Claude Lorrain, a French painter who worked mostly in Italy, painted landscapes that claim to be realistic, but in fact are extraordinarily simplified and easily interpreted in fractal terms. Historically, painters have always seen the possibilities of fractal structure, but it did not develop into a geometry, since very few wrote about it and probably none read about it.

HO: "Art happens", as stated by the 19th-century American painter James Whistler. That moment of fractal roughness "happened".

BM: Whistler was a great painter. So was the English painter Joseph Turner. His extraordinary wild images of ships burning in the sea perfectly combine Euclidean and fractal shapes. Eugene Delacroix, in his *Advice to a Young Painter*, which can be found in the artist's published letters, showed that he understood fractality intuitively, but at that time nobody could follow up.

I feel very privileged to have filled the role of bringing together phenomena that scientists had missed, and painters or photographers have implemented but never formalised.

HO: You told an interviewer that you have almost a collection, a Malraux kind of *musée imaginaire* of such artworks. How is this archive constituted?

BM: At this point it is mostly in my head^a an imaginary museum housing an imaginary collection of great paintings that come from different periods and styles but are linked by the artists' awareness of the splendid totality of fractal structures.

The museum would also have empty frames for cultures in which fractality was absent or negligible. For example, I think that the term 'Islamic' art is not useful, in fact is misleading, because Arabic art is not fractal, and Persian art very often is. Shiite and Sunni Muslims differ in many ways, including their art.

Some cultures have a very strong fractal aspect; Persian, Indian and Mughal architecture often show the contours of smaller domes within larger ones^a On the other hand, the Bauhaus was certainly anti-fractal with a passion. Perhaps this is why the Bauhaus has always been so unpopular, because humanity prefers architecture that offers a great deal of variety.

HO: That whole architectural movement of Max Taut, Hermann Finsterlin, the German expressionist group, was fractal. Not the Bauhaus.

BM: The Bauhaus reacted against the architectural complexity of late Beaux-Arts style; it worshipped the purity of the cube, and so on. Today, much of that is completely *passé*.

The first time I saw the Church of the Sagrada Familia in Barcelona, it had been reduced to two towers that Antoni Gaudi had himself designed; later I watched the process of its being completed, and was sorry. The two original towers had an extraordinary variety of detail, whereas the total monument as it now emerges is monotonous and decorative. The spirit of Gaudi was very simplistically rendered.

HO: Is contemporary architecture such as that by Zaha Hadid, or the deconstructivist Frank Gehry, fractal in nature?

BM: No. I find Gehry repetitive, though photographs of his Museo Guggenheim in Bilbao suggest that it is impressive. The relationships between the smooth masses of various sizes are fractal.

HO: In terms of urbanism, can one say that some cities are more fractal than others?

BM: I became interested in that aspect through someone I knew rather well, the late Yoshinobu Ashihara, a very successful architect in Tokyo.

HO: Was he part of Metabolism, the 1960s architectural movement that based its work on a theory of dynamic buildings and cities, rejecting the concept of fixed form and function, and instead advocating the creation of the whole by the accumulation of deconstructed, dissipated components?

BM: No. He was quite a traditional architect, in fact, an architect of the court. In one of his books, he compared the urban design of the Chinese or American city, both contained by square grids, and of Tokyo, a complicated city that includes chunks of everything; he felt that Tokyo's variety, novelty and unpredictability was generally not appreciated. Ashihara was a brilliant thinker, but his buildings were quite conventional.

HO: John Brockman, the editor and publisher of the online intellectual forum Edge in New York, told me about your near-mythical appearance at the Reality Club salon hosted by Edge, which he says was the most successful event he has ever organised. Have you had dialogues with visual artists, just as you have had with architects?

BM: Less so, unfortunately. That may reflect the problem visual art experiences at this point. It's not as great a period for artists as for architects. The installations demonstrate ingenuity but are very impermanent.

On the other hand, I have very strong connections with composers, who inhabit an entirely different world. In particular, György Ligeti came to me and confided that, until he saw my pictures, he had not understood an important aspect of music: it is not free to do as it pleases, because it must be fractal.

When Ligeti received a prize in New York, a major article appeared in which he listed the greatest designs ever. The list included the Book of Kells, the Taj Mahal^a and the Mandelbrot Set! That was an extremely strong statement, and I was very pleased to meet him shortly afterwards. We have had very interesting times together, including serious public discussions.

HO: I have never met him, but spoke to him once on the phone, some years ago. He had then said we would be meeting for an interview a few years hence because, as he put it, "I still have twenty years of music to write before one thinks of taking stock of all my works!"

BM: While he was a visiting composer at Yale for one or two weeks, a professor of piano played many of his pieces in his honour. I was dumbfounded by the quality of her playing. Till recently, Ligeti's piano repertoire was recognised as splendid but restricted to a few specialists. But this period was over. The lady was not a famous virtuoso but 'merely' a professor of piano at Yale's very good music school, yet she played Ligeti extremely well. And it was fascinating that Ligeti commented on his own works without the least self-indulgence, in fact, with remarkable ferocity. He said, for example, that to understand a certain piece one had to know that it had been written in the 1950s, during the time he worked in Darmstadt. So many talented musicians worked there that "one would do anything to get noticed!"

After he received the prize I mentioned earlier, he felt free to do as he pleased. What he wanted to do at that time was to write for piano; it was then that he really started to compose for this instrument.

HO: Ligeti and you are both somewhat in the same 'league', in terms of creativity.

BM: Thank you.

HO: Your great books are quite recent; and your work is ongoing. Yet several mathematicians have told me that the minds in this field produce their best at around 25 years of age^a The pressure to produce remarkable findings very quickly is a Damocles' sword hanging over their heads. You are more like a writer or a composer, in the sense that maturity brings about a certain evolution.

BM: The story that all mathematicians peak at 25 is a popular myth more than a reality. It is also a self-fulfilling prophecy, because of the fact that mathematicians who have not done anything remarkable before the age of 30 are pushed to the side and ignored. On the other hand, musicians continue indefinitely, but so have many mathematicians!

Some time back, I tried to draw up an informal list of those who have done their best work late in life. Alfred North Whitehead and Bertrand Russell published their celebrated *Principia Mathematicae* between 1910 and 1913. Therefore, Whitehead was about 50, and before the *Principia* had not produced anything of significance; Russell was just 30. Yet the cover page lists the co-authors in inverse alphabetic order, meant to emphasise that the senior author was Whitehead. He had been in charge of the mathematical part of the book but Russell was a famous high aristocrat, well known for pacifist views; he went to prison, etc. As a result, *Principia Mathematicae* is usually considered the work of Russell 'helped' by Whitehead, while the inverse would be more just.

Another deeply entrenched myth is that a mathematician who interrupts his work for 10 years "forgets everything", "loses his ability" and consequently is destined to disappear. Inversely, a writer or composer can stop for 10 years and come back stronger than before^a

HO: Can you describe your current theory of "negative dimension", on which you are planning to write a book?

BM: This topic seems completely eccentric, like a story from *Alice in Wonderland*, something like the Cheshire cat who vanishes leaving a smile that then vanishes gradually. But in fact this is a very serious topic I am investigating with a friend.

Eventually I hope to compile a small book covering everything I have written on this subject here and there, and the work of other, more recent, authors.

Increasingly negative dimensions characterise objects that become increasingly empty. At first glance this may appear as a kind of bad science fiction, but it is very practical and makes possible to attach a number to the notion of a "progressive emptiness", which elaborates on the common notion of simple emptiness.

It is a concept one should not romanticise, so I have recently decided to write of negative pro-dimension. It promises to become a new domain. Small or big^a? The future will tell.

HO: Do you have any utopian plans that you have not been able to see through?

BM: By doing 'one's best', one can do amazing things. I have a thousand plans at the same time.

My extremely complicated life has allowed two choices. One consists in setting priorities, staying committed to them, and doing everything possible to ensure that things are realised, in the order of priorities. The alternative policy judges, according to the circumstances, what is feasible and what is not, and then improvises with the means at one's disposal at a given moment. At IBM, I was absolutely systematic in following the non-systematic policy, which enabled me to do many things. I did not give a thought to seeking perfect, optimal conditions for seeing a plan through. I always concentrated on doing 'sufficiently well' *here and now*.

After retiring from IBM, I taught mathematics at Yale for 18 years. What was easy at IBM was difficult at Yale, and vice versa. I am keen to finish this work on negative dimensions; it has certainly started well. Also, befitting my age, I should not defer too long the completion of my memoirs. It must not be done too early, but one should not wait too long either.

HO: Time is everything^a

BM: Yes, especially in the domain of speculation and risk-taking. I am well into Volume I of my memoirs, and the remainder is sketched out. I do not know how many volumes I will write, but they will be different from each other. My memory is still very good, so I do everything from memory. When I am not sure, of course, I still verify! Nevertheless, it is very important for me to separate the things that are stored in my archives and in my memory. Later, if my memory starts to fail, I may be forced to privilege the archives.

I have led a complicated life, and observe various domains from various angles; hence I do not describe structures from a standard point of view. My perspectives are unconventional and multiple, and I believe I can contribute a little to history.

There is also the question of trans-disciplinarity, carried out with true contemporary geniuses. I have mentioned Ligeti; and there is another composer, Charles Wuorinen, with whom I did an extraordinary show titled "Music and Fractals" at the Guggenheim Museum in 1990. It is fascinating to see how two people from such different cultures can collaborate, if they desire to do so.

My book *The (Mis)behaviour of Markets: A Fractal View of Risk, Ruin and Reward*, published in 2004, was difficult to write. My collaborator Richard Hudson, managing editor of *The Wall Street Journal*, was a student of mathematics prior to becoming a journalist; this made the communication of certain ideas easier. He wrote the body of

the text; next, I verified everything and we discussed the text a thousand times. This book is reaching out to a wide public. It complements my earlier book *Fractals and Scaling in Finance: Discontinuity, Concentration, Risk*, which demanded our competence in mathematics; this was published by Springer in 1999.

HO: The notion of risk explored in your book on markets leads me to ask if it is possible to have a 'fractal' view of the art market.

BM: Absolutely. The inequality of prices on the art market is astounding: from tens of millions of dollars to only a few dollars. I became deeply interested in the paintings of Frantisek Kupka, the first avant-garde Czech painter, because a certain period of his work was clearly 'fractal'. A major part of Kupka's paintings belonged to a Jewish banker in Prague who had financially supported Kupka, disregarding the fact that the artist was anti-Semitic. Those paintings, first confiscated by the Nazis and then by the Communists, have returned to his heirs. But export restrictions affect the market, and another patron, a mysterious German lady, also owns many of his paintings and can clearly influence their value.

HO: You told an interviewer that in the film *Star Trek 2*, there was a slightly swirling multi-fractal method, used to create some special effects. Did the filmmakers consult you?

BM: Not at all! To my disappointment, this film nowhere credited my scientific publications. *Star Trek 2* was made by Lucas Films; they simply bought my 1977 book, *Fractals: Form, Chance and Dimension*, which was not an extravagant investment for them! [Laughs]. The mountains in the film were made using a variant of my method. When the film was first released in California, I did not go to the pre-release show, simply because I did not think that it was worth the travel. However, when it came to my neighbourhood, something quite surprising took place. One of my assistants who had seen the film conveyed the bad news that the special effects created with fractals had been edited out. The next day my wife and I went to the film, and the fractals were staring at both of us. My assistant had been misled by the realistic treatment; he had not seen the fractals, as he was still not used to them.

Every graphic design class teaches fractals, and in commercials it is a commonplace technique, an unnoticed daily application⁸

HO: The contemporary writer Fernando Arrabal mentions you in almost all his interviews, especially when it comes to the subject of fate and memory.

BM: Arrabal has written many articles on me, based on his own observations and on interviews that he has read. In addition, many of his recent novels have realistic characters who seem to be inspired by my personality. I do not read all the books that have a character who resembles me. I prefer novels that feature my mathematical work, which usually means the Mandelbrot Set. In 1990, the well-known science fiction author Arthur C. Clarke published a book based on the Mandelbrot Set. Titled *The Ghost from the Grand Banks*, it is about an expedition to find the wreck of the Titanic. The cover shows a Mandelbrot Set with the Titanic sinking in the centre. Very nice.

HO: In the introduction of your book on markets, Richard Hudson says it is written a bit like a polyphonic opera with an interplay of voices. Do you agree?

BM: I am passionate about opera. There was a time when I went every second day to the opera house and spent many hours a day listening to opera records. I find that it is one of the more powerful modes of expression. I am a great admirer of Mozart and Verdi; I like Rossini, but not Wagner that much. In the 19th century, opera was understood as a very popular way of writing. Ask people today if the 20th century has been good to opera, and they will probably say no. Today, opera is widely considered an 'outdated', 'obsolete' form of art that requires justification and explanation. This is completely false! Several of the most extraordinary operas in the history of music were written in the 20th century. *Wozzeck*, almost horribly simple, is a stunning instance that grew from an odd specialty almost to the centre of the repertoire. So is Francis Poulenc's remarkably melodramatic and powerful *Les Dialogues des Carmélites* (Dialogues of the Carmelites).

HO: Would you consider *Wozzeck*, the famous 1925 atonal opera by the expressionist composer Alban Berg, and works by Verdi, to be 'fractal' operas?

BM: There is surely a fractal element, as opera has to reveal various levels for it to be effective; and there are so many elements one can connect. What I find truly interesting in opera is^a the opera itself! I love the human voice.

Hudson was probably implying that it is important in scientific work to take a view and a presentation which is 'active'; and nothing is more active than an opera! Many scientific articles are completely flat because they are written for people who do not have to be convinced. They are part of a small circle within a well-established domain; they write for each other, know more or less everybody, or are introduced by their thesis supervisors or mentors. As a result, style is a very secondary and unimportant thing for them. In my case, the fact that I write for an unknown public necessarily influences and shapes my style. Whether it is opera or Greek drama, one must know how to enter into a subject quickly because one cannot assume that the public will wait to understand. One has to be able to speak to people in their style, motivate and perhaps amuse the reader a little.

HO: Paraphrasing the title of Rainer Maria Rilke's magnificent 1903 text *Letters to a Young Poet*, do you, in 2005, have advice for a young researcher or a young mathematician?

BM: This has become very difficult^a In Rilke's time, poetry was a fully defined form of art. The problem with science at the moment is the rapidity of that change. By good fortune I managed to work in science 'solo', with a restricted number of collaborators. I have never worked on an article with more than two or a few authors. This makes me a rare exception. Most science is very different today. Hundreds of people join in writing an article, and their work leans on a laboratory employing a thousand people^a When young people come and ask for advice, I have to say that both my personal and informal way of practicing science was already difficult in my time, but now it has become inconceivable. One has to be very flexible, more than before. My only advice would be to always keep your options open, because it is possible that you might have to change domains^a

HO: There are many anecdotal stories about well-known mathematicians and scientists such as Jacques Hadamard and Geoffrey Harold Hardy, whose critical ideas came to them intuitively, in unexpected circumstances. This has led to discussions about the nature and sources of our knowledge. My scientist friend Israel Rosenfield asks about your view of knowledge, how the brain works, your ideas about memory, perception and consciousness^a Is your approach to this reflected in your own work?

BM: I've skimmed through Hadamard's book, but never read it properly; anyway, it was written late in his life, with much help from his daughter. I've also read *The Mathematician's Apology* by Hardy. Indeed, the question of chance is very disturbing. If I not had paid attention to those pictures, my science would have been very different, and I might have become a different person. Perhaps ; but this is not for me to judge ; science would have progressed differently.

Hadamard was a wise and balanced man. Hardy was an ideologue with ideas about 'pure mathematics' that I consider ridiculous; yet these ideas keep being revived, then collapse, and so on. The canonical example Hardy gave of 'pure mathematics', with no applications outside of the self-interest of studying the topic, was number theory in general, and the study of prime numbers in particular. Number theorists such as Hardy took pride in doing work that had absolutely no military significance. However, this vision was shattered when in the 1970s prime numbers became the basis for the creation of public key cryptography algorithms. This is a use that Hardy, a pacifist, would have hated.

In relation to the reality of mathematics, I am a forceful moderate, not an ideologue. When discovering the Mandelbrot Set, I had absolutely no impression of inventing it. While nobody had seen it before, I had a very strong feeling that it existed but remained hidden because nobody had the insight to identify it. All that is actually a great mystery. Mathematics and music are crucial achievements of humanity, and continue to expand in directions quite different from what the ideologues expect.

HO: Alighiero Boetti, the artist we discussed earlier, repeatedly turned to your work; for him, it was an example of resistance against the homogenising forces of globalisation, since your perspectives foreground variety, diversity, complexity. Today we are in a very different moment, of a rapidly expanding and intensifying globalisation. What is your point of view here, in relation to your study of markets?

BM: This question is far too complicated to be answered in a few words, so I prefer not to try. I don't think globalisation necessarily decreases variety, but this is a tentative opinion.

I enjoy diversity enormously. I favoured it in my choice of topics to pursue, and I still do my best to help it increase rather than vanish.

HO: A wonderful conclusion. Thank you very much.

Benoit Mandelbrot, in his long career, has relied on the good eye to interpret mathematics in its widest scope. At its purest end, he phrased a number of new and extremely difficult conjectures, grounded in interpretation of computer graphics. He contributed to many sciences from physics to finance, often solving down-to-earth questions by using reputedly esoteric mathematics. He has drawn bridges over the chasms that separate mathematics, science and technology from one another and from the interests of the common man and the child. His life work – the fractal geometry of roughness in nature and culture – has been acknowledged by the 1993 Wolf Prize for Physics (which hailed him for having "changed our view of nature"), the 2003 Japan Prize for Science and Technology, and many other awards. He is IBM Fellow Emeritus, Sterling Professor Emeritus of Mathematical Sciences at Yale University, and Battelle Fellow.
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